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# HL Paper 3

Let  $c$  be a positive, real constant. Let  $G$  be the set  $\{x \in \mathbb{R} \mid -c < x < c\}$ . The binary operation  $*$  is defined on the set  $G$  by  $x * y = \frac{x+y}{1+\frac{xy}{c^2}}$ .

- a. Simplify  $\frac{c}{2} * \frac{3c}{4}$ . [2]
- b. State the identity element for  $G$  under  $*$ . [1]
- c. For  $x \in G$  find an expression for  $x^{-1}$  (the inverse of  $x$  under  $*$ ). [1]
- d. Show that the binary operation  $*$  is commutative on  $G$ . [2]
- e. Show that the binary operation  $*$  is associative on  $G$ . [4]
- f. (i) If  $x, y \in G$  explain why  $(c-x)(c-y) > 0$ . [2]
- (ii) Hence show that  $x + y < c + \frac{xy}{c}$ .
- g. Show that  $G$  is closed under  $*$ . [2]
- h. Explain why  $\{G, *\}$  is an Abelian group. [2]
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A random variable  $X$  has probability density function

$$f(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{2} & 0 \leq x < 1 \\ \frac{1}{4} & 1 \leq x < 3 \\ 0 & x \geq 3 \end{cases}$$

- a. Sketch the graph of  $y = f(x)$ . [1]
- b. Find the cumulative distribution function for  $X$ . [5]
- c. Find the interquartile range for  $X$ . [3]
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